Decentralized Finance

Privacy on the Blockchain

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Can we have private transactions on a public blockchain?

Naïve reasoning:

universal verifiability $\Rightarrow$ transaction data must be public.

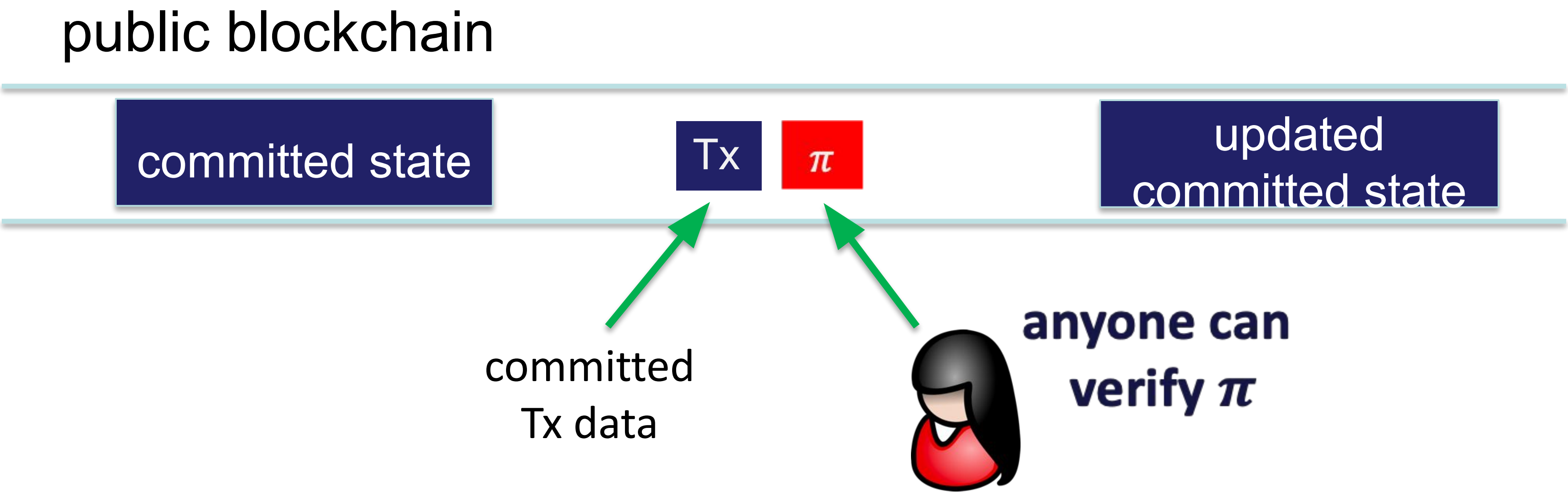
otherwise, how we can verify Tx ??

Goal for this lecture:

crypto magic $\Rightarrow$ private Tx on a publicly verifiable blockchain

Crypto tools: **commitments** and **zero knowledge proofs**

Private Tx with universal verifiability: how?



(reveals nothing about Tx data or state)

Committed data: short (hiding) commitment on chain

Proof π: succinct *zero-knowledge proof* that

(1) committed Tx data is consistent with committed current state, and

(2) committed new state is correct

The need for privacy in the financial system

- Supply chain privacy:

  A car company does not want to reveal
  how much it pays its supplier for tires, wipers, etc.

- Payment privacy:
  - A company that pays its employees in crypto needs to keep list of employees and their salaries private.
  - Privacy for rent, donations, purchases

- Business logic privacy:

  Can the code of a smart contract be private?

Types of Privacy

- **Pseudonymity: (weak privacy)**
  - One consistent pseudonym (e.g. reddit)
    - Pros: Reputation
    - Cons: Linkable posts: one post linked to you $\Longrightarrow$ all posts linked to you

- **Full anonymity:**
  - User's transactions are unlinkable
    - The system cannot tell if two transactions are from the same person
  - Maintaining reputation is possible but more complex

Privacy in Ethereum?

- Accounts:
  - Every account balance is public
  - For Dapps: code and internal state are public
  - All account transactions are linked to account

<u>etherscan.io:</u>

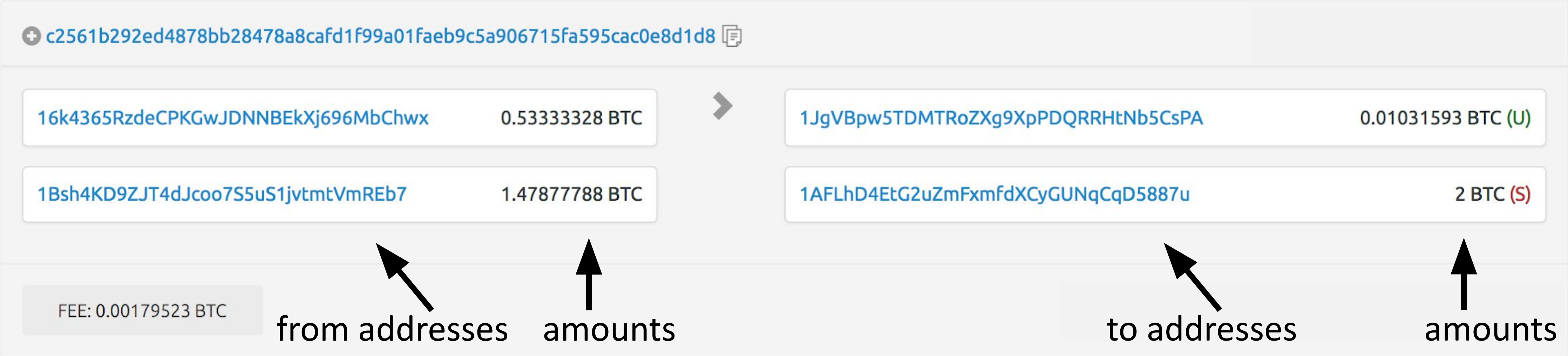
Address 0x1654b0c3f62902d7A86237

…

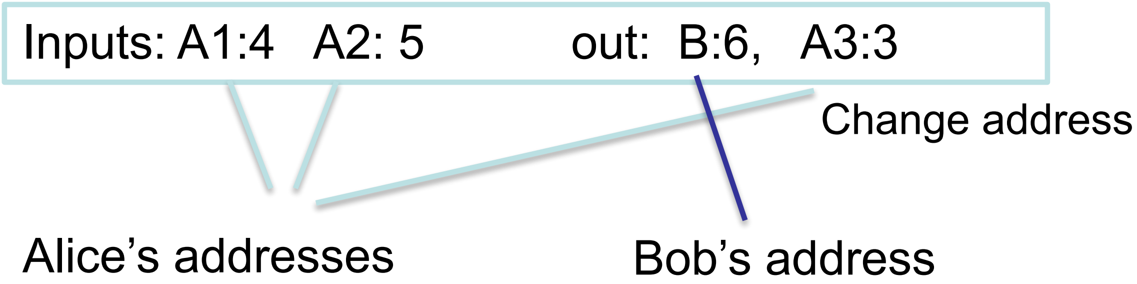
Balance:	1.114479450024297906 Ether
Ether Value:	$4,286.34 (@ $3,846.05/ETH)

Txn Hash	Method ⓘ	Block
0x0269eff8b4196558c07…	Set Approval For...	13426561
0xa3dacb0e7c579a99cd…	Cancel Order_	13397993
0x73785abcc7ccf030d6a…	Set Approval For...	13387834
0x1463293c495069d61c…	Atomic Match_	13387703

Privacy in Bitcoin?



Alice can have many addresses (creating address is free)



Transaction data can be used to link addresses to a single owner and to a physical entity

(chainalysis)

Privacy of Digital Payments

Payments publicly visible and linkable

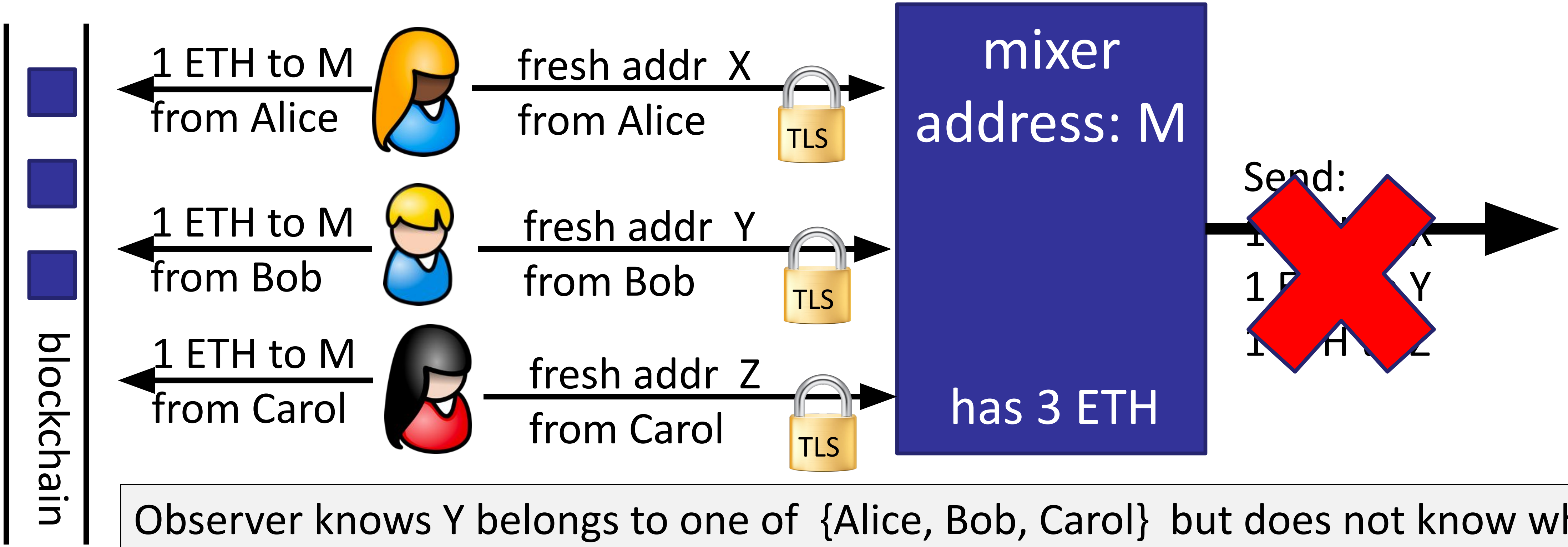
Payments only visible to bank

private payments

Less private

More private

Simple blockchain anonymity via mixing



Observer knows Y belongs to one of {Alice, Bob, Carol} but does not know which one

⟹ anonymity set of size 3.

⟹ Bob can mix again with different parties to increase anonymity set.

Problems: (i) mixer knows all, (ii) mixer can abscond with 3 ETH !!

Mixing without a mixer? on Bitcoin: **CoinJoin** (e.g., Wasabi), on Ethereum: **Tornado cash**

Negative aspects of privacy in finance

Criminal activity:

- Tax evasion, ransomware, …

Can we support positive applications of private payments, but prevent the negative ones?

- Can we ensure legal compliance while preserving privacy?
- Yes! With proper use of zero knowledge proofs

Next segment: commitments

An important tool

Cryptographic Commitments

https://defi-learning.org/

Cryptographic commitments

Cryptographic commitment: emulates an envelope

Many applications: e.g., a DAPP for a sealed bid auction

- Every participant **commits** to its bid,
- Once all bids are in, everyone opens their commitment

Cryptographic Commitments

Syntax: a commitment scheme is two algorithms

- **commit**(*msg*, *r*) → *com*

  secret randomness in R — commitment string

- **verify**(*msg*, *com*, *r*) → accept or reject

  anyone can verify that commitment was opened correctly

Commitments: security properties

- **binding**: Bob cannot produce two valid openings for ***com***.

  More precisely: no efficient adversary can produce

  com, (m_1, r_1), (m_2, r_2)

  such that verify(m_1, ***com***, r_1) = verify(m_2, ***com***, r_2) = accept

  and $m_1 \neq m_2$.

- **hiding**: ***com*** reveals nothing about committed data

  commit(m, r) → ***com***, and r is sampled uniformly in R,

  then ***com*** is statistically independent of m

Example 1: hash-based commitment

Fix a hash function $H: M \times R \rightarrow C$ (e.g., SHA256)

where H is collision resistant, and $|R| \gg |C|$

- commit($m \in M$, $r \leftarrow R$): $\boldsymbol{com} = H(m, r)$
- verify(m, $\boldsymbol{com}$, r): accept if $\boldsymbol{com} = H(m, r)$

binding: follows from collision resistance of H

hiding: follows from a mild assumption on H

Example 2: Pedersen commitment

G = finite cyclic group = $\{1, g, g^2, \ldots, g^{q-1}\}$ where $g^i \cdot g^j = g^{(i+j \bmod q)}$

$q = |G|$ is called the **order** of G. Assume q is a prime number.

Fix g, h in G and let $R = \{0, 1, \ldots, q-1\}$. For $m, r \in R$ define

$$H(m, r) = g^m \cdot h^r \in G$$

Fact: for a "cryptographic" group G, this H is collision resistant.

⇒ commitment scheme: **commit** and **verify** as in example 1

$$\textbf{commit}(m \in R,\ r \leftarrow R) = H(m, r) = g^m \cdot h^r$$

An interesting "homomorphic" property

- $\textbf{commit}(m \in R,\ r \leftarrow R) = H(m,r) = g^m \cdot h^r$

Suppose: $\text{commit}(m_1 \in R,\ r_1 \leftarrow R) \rightarrow com_1$

$\text{commit}(m_2 \in R,\ r_2 \leftarrow R) \rightarrow com_2$

Then: $\text{com}_1 \times \text{com}_2 = g^{m1+m2} \cdot h^{r1+r2} = \text{commit}(\underline{m_1+m_2},\ r_1+r_2)$

⇒ anyone can sum committed value

Next segment: zero knowledge proofs

An important privacy tool

What is a zk-SNARK?

Succinct Non-interactive ARgument of Knowledge

zk-SNARK: Blockchain Applications

Scalability:

- SNARK Rollup (zk-SNARK for privacy from public)

Privacy: Private Tx on a public blockchain

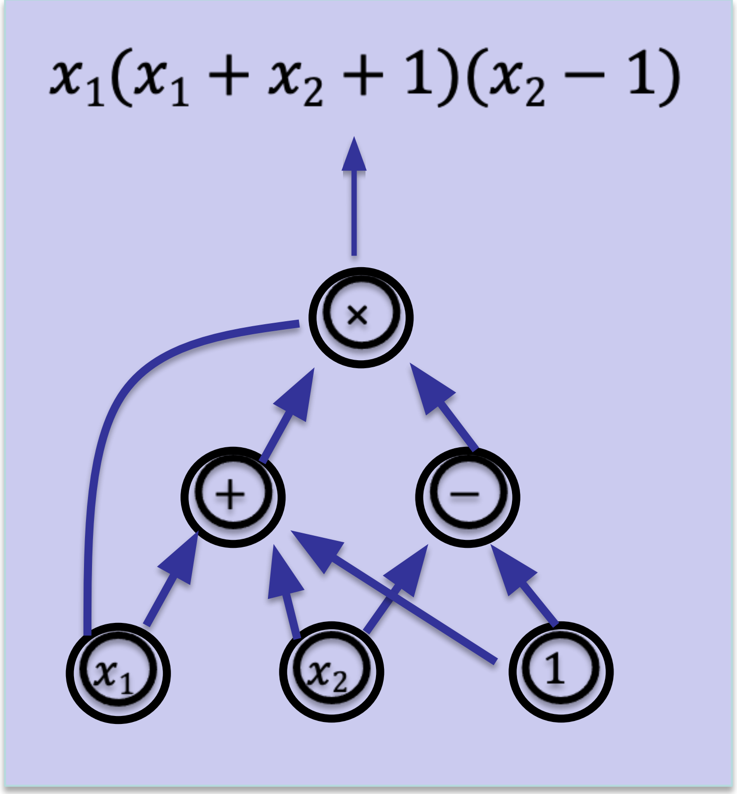
- Confidential transactions
- Tornado cash
- Private Dapps: Aleo

Compliance:

- Proving solvency in zero-knowledge
- Zero-knowledge taxes

(1) arithmetic circuits

- Fix a finite field $\mathbb{F} = \{0, \ldots, p-1\}$ for some prime p>2.
- **Arithmetic circuit:** $C: \mathbb{F}^n \rightarrow \mathbb{F}$
  - directed acyclic graph (DAG) where
    - internal nodes are labeled +, −, or ×
    - inputs are labeled $1, x_1, \ldots, x_n$
  - defines an n-variate polynomial with an evaluation recipe
- $|C| =$ # gates in C



Interesting arithmetic circuits

Examples:

- $C_{hash}(h, \mathbf{m})$: outputs 0 if $SHA256(\mathbf{m}) = h$, and $\neq 0$ otherwise

  $C_{hash}(h, \mathbf{m}) = \big(h - SHA256(\mathbf{m})\big)$, $|C_{hash}| \approx$ 20K gates

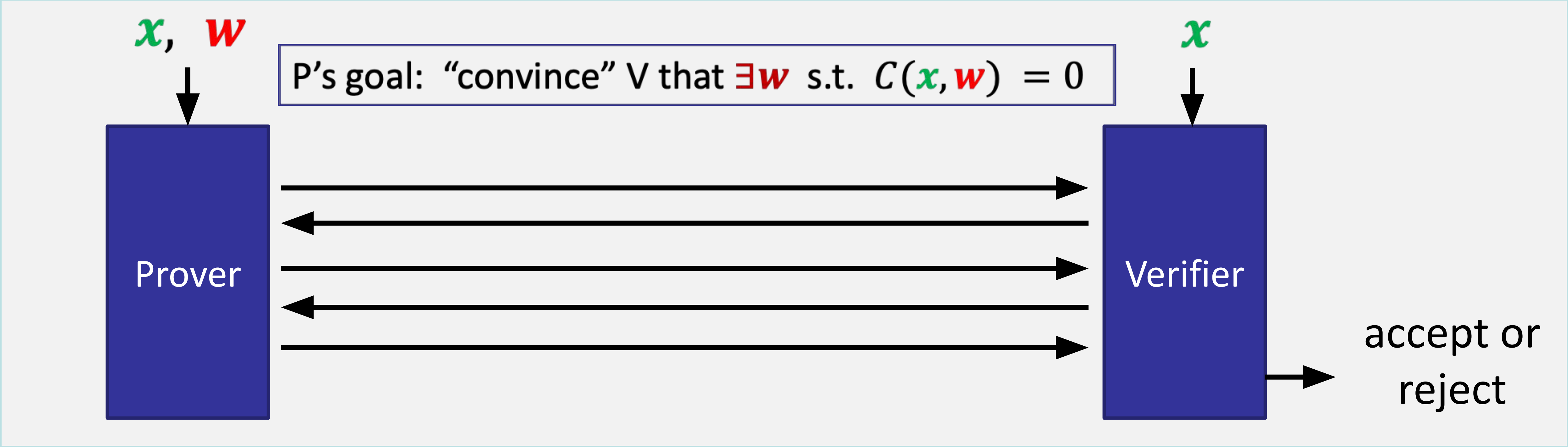
- $C_{sig}(pk, m, \sigma)$: outputs 0 if σ is a valid ECDSA signature on m with respect to pk

(2) Argument systems (for NP)

Public arithmetic circuit: $C(x, w) \to \mathbb{F}$

public statement in $\mathbb{F}^n$ — x

secret witness in $\mathbb{F}^m$ — w



Two types of argument systems: interactive vs. non-interactive

Interactive: proof takes multiple P↔V rounds of interaction

- Useful when there is a single verifier, e.g. a compliance auditor
- Example: zero-knowledge proof of taxes to tax authority

Non-interactive: prover sends a <u>single</u> message (proof) to verifier

- Used when many verifiers need to verify proof, e.g., Rollup systems
- SNARK: short proof that is fast to verify

(non-interactive) Preprocessing argument system

Public arithmetic circuit: $C(x, w) \rightarrow \mathbb{F}$

public statement in $\mathbb{F}^n$ (x)

secret witness in $\mathbb{F}^m$ (w)

Preprocessing (setup): $\mathbf{S}(C) \rightarrow$ public parameters (S_p, S_v)

Preprocessing argument System

A non-interactive argument system is a triple (S, P, V):

- **S**(C) → public parameters (S_p, S_v) for prover and verifier
- **P**$(S_p, \boldsymbol{x}, \boldsymbol{w})$ → proof π
- **V**$(S_v, \boldsymbol{x}, \boldsymbol{\pi})$ → accept or reject

Argument system: requirements (informal)

Prover $P(S_p, \mathbf{x}, \mathbf{w})$ — Verifier $V(S_v, \mathbf{x}, \boldsymbol{\pi})$

proof π →

accept or reject

Complete: $\forall x, w$: $C(\mathbf{x}, \mathbf{w}) = 0 \Rightarrow \Pr[\ V(S_v, x, P(S_p, \mathbf{x}, \mathbf{w})) = \text{accept}\ ] = 1$

Argument of knowledge: V accepts $\Rightarrow$ P "knows" $\mathbf{w}$ s.t. $C(\mathbf{x}, \mathbf{w}) = 0$

P* does not "know" $\mathbf{w}$ $\Rightarrow$ $\Pr[\ V(S_v, x, \pi) = \text{accept}\ ] <$ negligible

Optional: **Zero knowledge**: $(S_v, \mathbf{x}, \pi)$ "reveals nothing" about $\mathbf{w}$

Preprocessing **SNARK**

A **succinct non-interactive argument system** is a triple (S, P, V):

- **S**(C) → public parameters (S_p, S_v) for prover and verifier
- **P**$(S_p, \boldsymbol{x}, \boldsymbol{w})$ → short proof π ; $|\pi| = O(\mathbf{log}(|\boldsymbol{C}|), \lambda)$
- **V**$(S_v, \boldsymbol{x}, \boldsymbol{\pi})$ → accept or reject ; time(V) = $O(|x|, \mathbf{log}(|\boldsymbol{C}|), \lambda)$

short "summary" of circuit

Why preprocess C??

Preprocessing **SNARK**

A <u>**succinct**</u> **non-interactive argument system** is a triple (S, P, V):

- **S**(C) → public parameters (S_p, S_v) for prover and verifier
- **P**(S_p, $\boldsymbol{x}$, $\boldsymbol{w}$) → <u>short</u> proof π ; $|\pi| = O(\, \mathbf{log}(|\boldsymbol{C}|)\,,\ \lambda)$
- **V**(S_v, $\boldsymbol{x}$, $\boldsymbol{\pi}$) → accept or reject ; time(V) = $O(|x|,\ \mathbf{log}(|\boldsymbol{C}|)\,,\ \lambda)$

If (S, P, V) is **succinct** and **zero-knowledge** then we say that it is a **zk-SNARK**

The trivial argument system

(a) Prover sends $\boldsymbol{w}$ to verifier,

(b) Verifier checks if $C(\boldsymbol{x}, \boldsymbol{w}) = 0$ and accepts if so.

Problems with this:

(1) $\boldsymbol{w}$ might be secret: prover does not want to reveal $\boldsymbol{w}$ to verifier

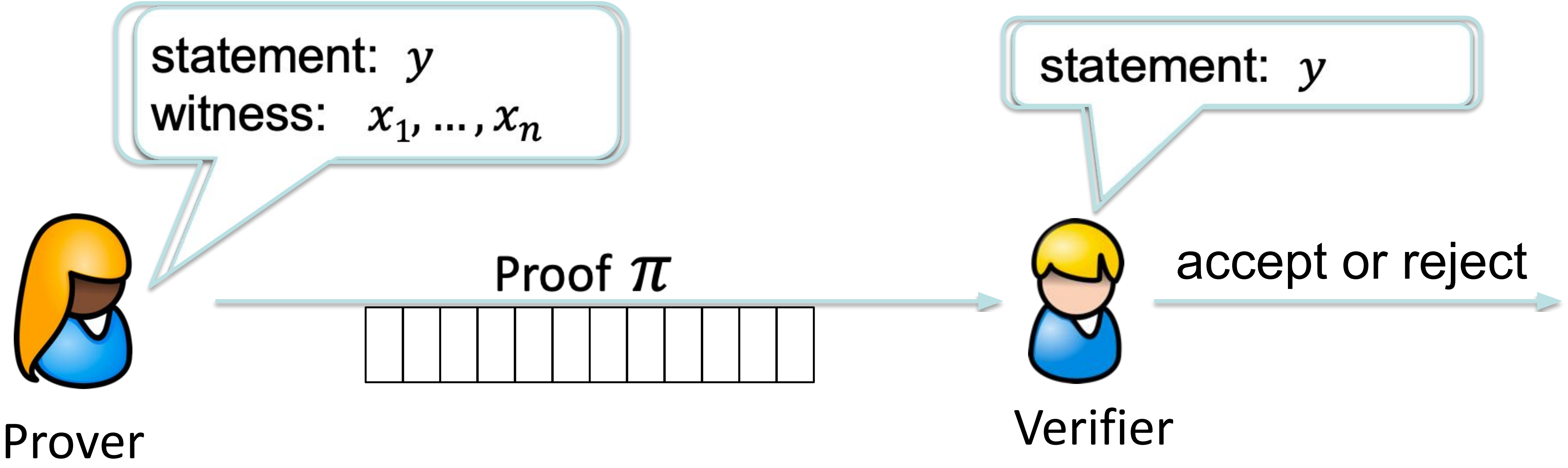
(2) $\boldsymbol{w}$ might be long: we want a “short” proof

(3) computing $C(\boldsymbol{x}, \boldsymbol{w})$ may be hard: we want a “fast” verifier

An example

Prover: I know $(x_1, \dots, x_n) \in X$ such that $H(x_1, \dots, x_n) = y$

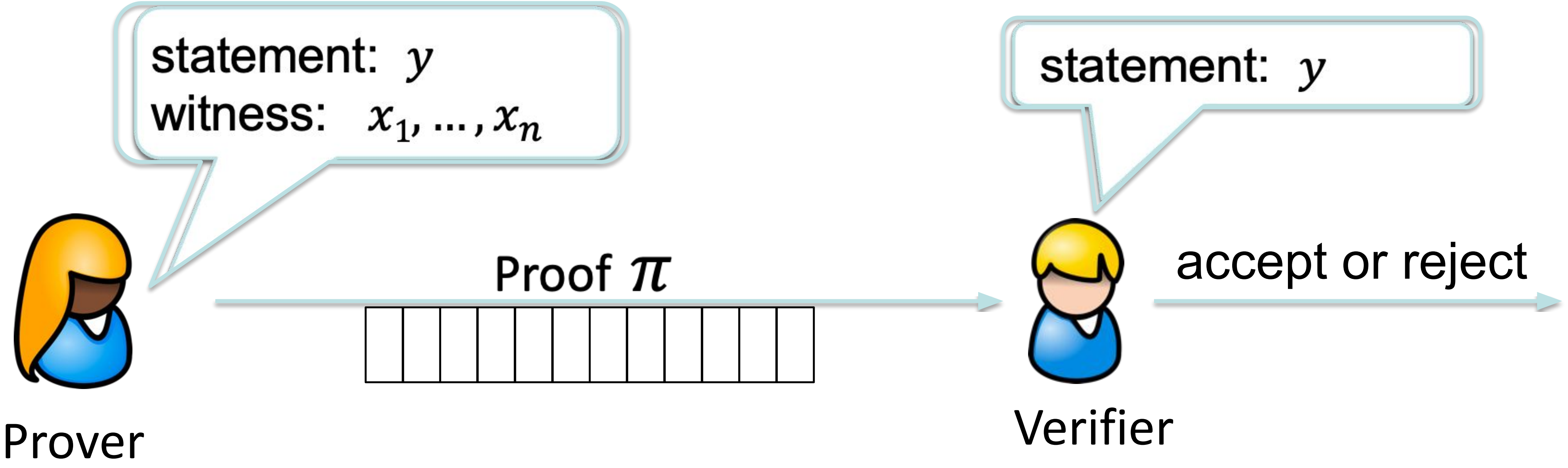
SNARK: $\mathrm{size}(\pi)$ and $\mathrm{VerifyTime}(\pi)$ is $O(\log n)$!!



An example

How is this possible ???

SNARK: $\text{size}(\pi)$ and $\text{VerifyTime}(\pi)$ is $O(\log n)$!!



Types of preprocessing Setup

Recall setup for circuit C: **S**(C) → public parameters (S_p, S_v)

Types of setup:

trusted setup per circuit: **S**(C) uses data that must be kept secret

compromised trusted setup ⇒ can prove false statements

trusted but universal (updatable) setup: secrets in **S**(C) are independent of C

$$\boldsymbol{S} = (S_{init}, S_{pre}): \quad \underbrace{S_{init}(\lambda) \to U,}_{\text{one-time}} \quad \underbrace{S_{pre}(U, C) \to (S_p, S_v)}_{\text{no secret data}}$$

transparent setup: **S**(C) does not use secret data (no trusted setup)

better

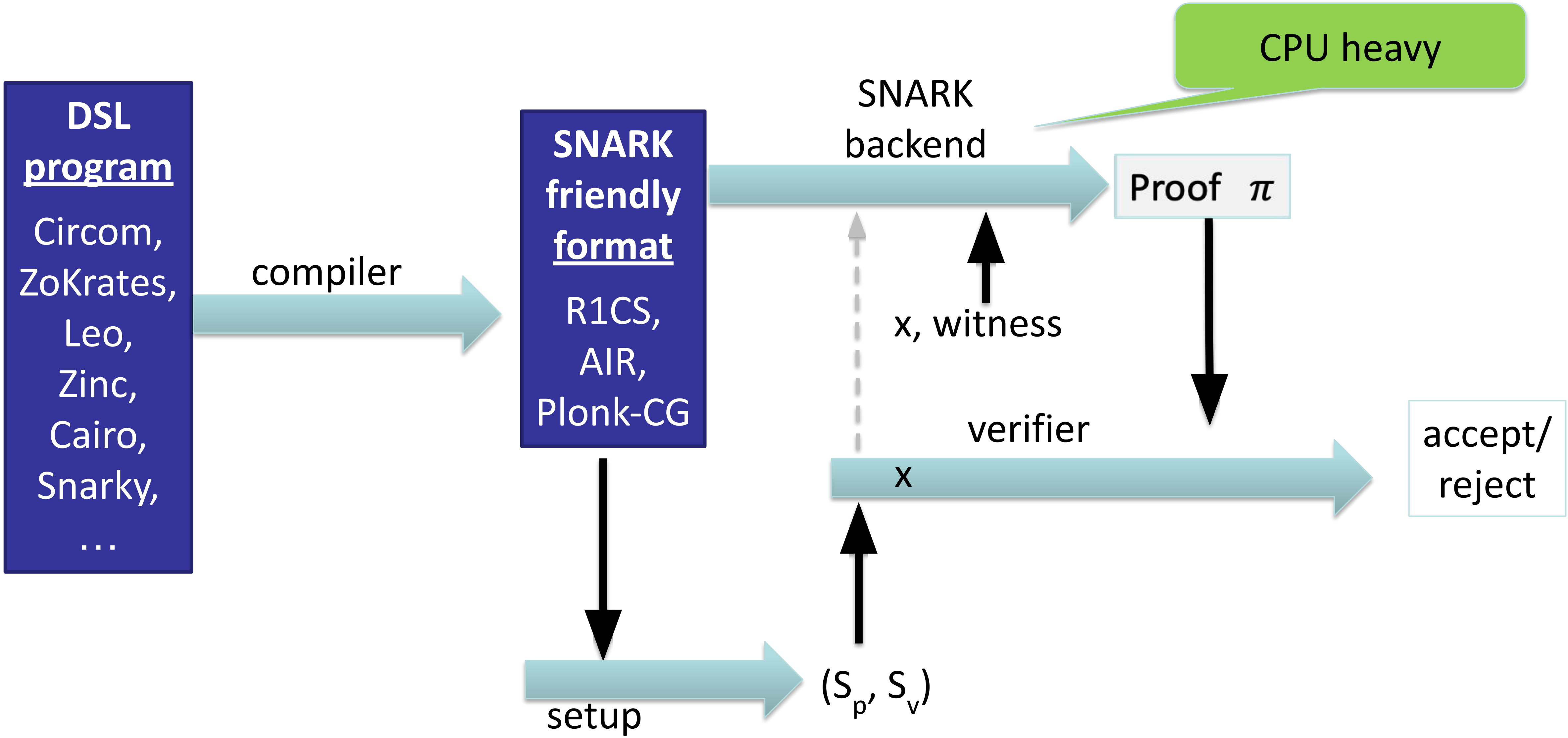
Significant progress in recent years

- **Kilian'92, Micali'94**: succinct transparent arguments from PCP
  - impractical prover time
- **GGPR'13, Groth'16, …**: linear prover time, **constant size proof** $(O_\lambda(1))$
  - **trusted setup per circuit** (setup alg. uses secret randomness)
  - compromised setup ⇒ proofs of false statements
- **Sonic'19, Marlin'19, Plonk'19**, … : universal trusted setup
- **DARK'19, Halo'19, STARK**, … : no trusted setup (transparent)

Types of SNARKs (partial list)

		size of S_p	**verifier time**	**trusted setup?**
Groth'16	O(1)		O(1)	yes/per circuit
Plonk/Marlin	O(1)		O(1)	yes/universal
Bulletproofs		O(1)		no
STARK		O(1)		no
DARK		O(1)		no
⋮	⋮			⋮

A SNARK software system



ZoKrates Example

Goal: prove knowledge of a hash (SHA256) preimage for a given $x \in \{0,1\}^{256}$

- For a public x, prover knows $w \in \mathbb{F}_p$ such that SHA256(w) = x
- $\mathbb{F}_p$ is a 254-bit prime field

Compiled into an arithmetic circuits (R1CS) over $\mathbb{F}_p$

```
def main(field x[2],   private field w) -> (field):
   h = sha256packed( w )
   h[0] == x[0]       //  check top 128 bits
   h[1] == x[1]       //  check bottom 128 bits
   return 1
```

How to define “argument of knowledge” and “zero knowledge”?

Definitions: (1) argument of knowledge

Goal: if V accepts then P "knows" w s.t. $C(x, w) = 0$

What does it mean to "know" w ??

informal def: P knows w, if w can be "extracted" from P

P

Definitions: (1) argument of knowledge

Formally: (S, P, V) is an argument **of knowledge** for a circuit C if for every poly. time adversary $A = (A_0, A_1)$ such that

$$S(C) \to (S_p, S_v), \quad (x, st) \leftarrow A_0(S_p), \quad \pi \leftarrow A_1(S_p, x, st):$$

$$\Pr[V(S_v, x, \pi) = \text{accept}] > 1/10^6 \qquad \text{(non-negligible)}$$

there is an efficient **extractor** E (that uses A_1 as a black box) s.t.

$$S(C) \to (S_p, S_v), \quad (x, st) \leftarrow A_0(S_p), \quad \boxed{w \leftarrow E^{A_1(S_p, x, st)}(S_p, x)}:$$

$$\Pr[C(x, w) = 0] > 1/10^6 \qquad \text{(non-negligible)}$$

If holds for **all** A, then (S, P, V) is a **proof of knowledge**.

Definitions: (2) Zero knowledge (against an honest verifier)

(S, P, V) is **zero knowledge** if for every $x \in \mathbb{F}^n$

proof π "reveals nothing" about ***w***, other than its existence

What does it mean to "reveal nothing" ??

Informal def: π "reveals nothing" about ***w*** if the verifier can generate π **by itself** $\Longrightarrow$ it learned nothing new from π

- (S, P, V) is **zero knowledge** if there is an efficient alg. **Sim** s.t. $(S_p, S_v, \pi) \leftarrow$ ***Sim***(C, x) "look like" the real S_p, S_v and π.

Main point: ***Sim***(C,x) simulates π without knowledge of ***w***

Definitions: (2) Zero knowledge (against an honest verifier)

Formally: (S, P, V) is (honest verifier) **zero knowledge** for a circuit C

if there is an efficient simulator ***Sim*** such that

for all $x \in \mathbb{F}^n$ s.t. $\exists w: C(x, w) = 0$ the distribution:

> (S_p, S_v, x, π): where $(S_p, S_v) \leftarrow S(C)$, $\pi \leftarrow P(S_p, x, \boldsymbol{w})$

is indistinguishable from the distribution:

> (S_p, S_v, x, π): where $(S_p, S_v, \pi) \leftarrow \boldsymbol{Sim}(C, x)$

How to build a zk-SNARK?

Recall: A zero knowledge preprocessing argument system.

Prover generates a **short** proof that is **fast** to verify

How to build a zk-SNARK ??

Not in this course …

(see, e.g., cs251)

Next segment: confidential transactions

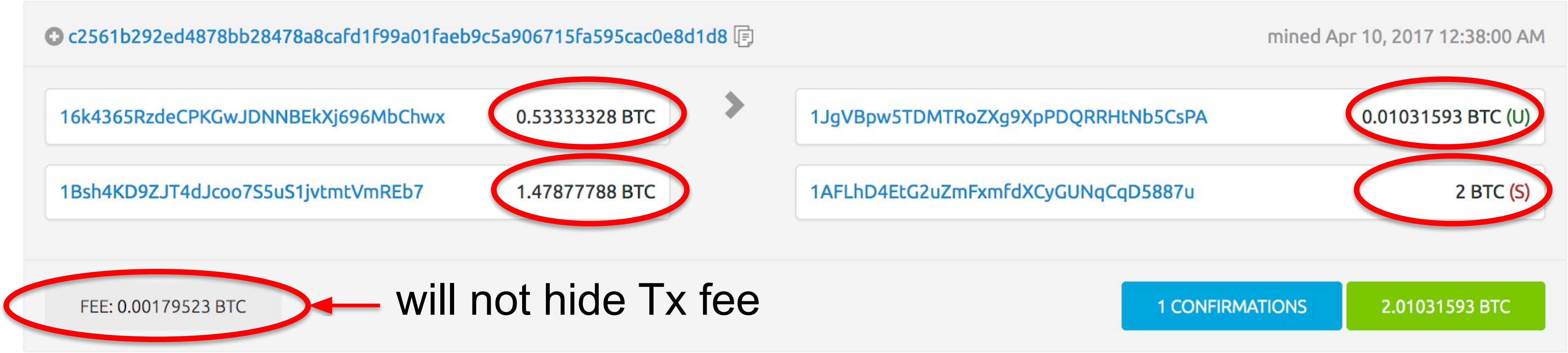
Private Tx Warmup: Confidential Transactions

https://defi-learning.org/

Confidential Transactions (CT)

Current Bitcoin Tx expose full payment details:

$\Longrightarrow$ Businesses cannot use Bitcoin for supply chain payments, salaries, etc.



Goal: hide amounts in Bitcoin transactions.

Confidential Tx: how?

Bitcoin Tx today: AcmeCo: **30** → Alice: **1**, AcmeCO: **29**

8 bytes

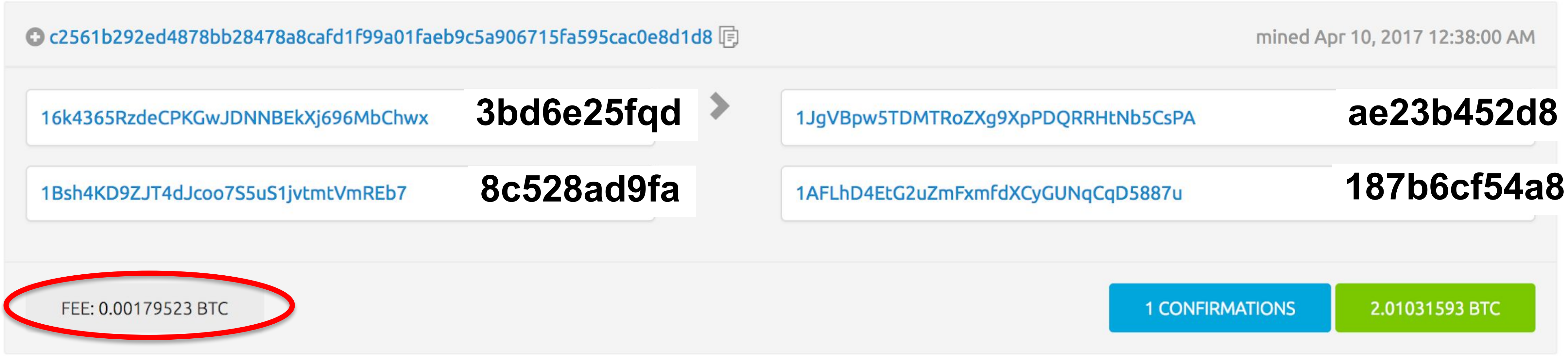
The plan: replace amounts by commitments to amounts

AcmeCo: $\mathbf{com_1}$ → Alice: $\mathbf{com_2}$, AcmeCo: $\mathbf{com_3}$

32 bytes

where $\mathbf{com_1}$ = commit(30, r_1), $\mathbf{com_2}$ = commit(1, r_2), $\mathbf{com_3}$ = commit(29, r_3)

Now blockchain hides amounts



How much was transferred ???

The problem: how can miners verify Tx?

AcmeCo: $\mathbf{com_1}$ → Alice: $\mathbf{com_2}$, AcmeCo: $\mathbf{com_3}$

$\mathbf{com_1}$ = commit(m_1=30, r_1), $\mathbf{com_2}$ = commit(m_2=1, r_2), $\mathbf{com_3}$ = commit(m_3=29, r_3)

Solution: zk-SNARK (special purpose, optimized for this problem)

- AcmeCo: (1) privately send r_2 to Alice

  (2) construct a proof π for

  statement = x = ($\mathbf{com_1}$, $\mathbf{com_2}$, $\mathbf{com_3}$, **Fees**)

  witness = w = (m_1, r_1, m_2, r_2, m_3, r_3)

  where circuit C(x,w) outputs 0 iff:

CT arithmetic circuit:

(i) $\mathbf{com_i}$ = commit(m_i, r_i) for i=1,2,3,

(ii) $m_1 = m_2 + m_3$ + Fees,

(iii) $m_2 \geq 0$ and $m_3 \geq 0$

The problem: how can miners verify Tx?

AcmeCo: (1) privately send r_2 to Alice

(2) construct a ZK proof π that Tx is valid

(3) embed π in Tx (need short proof! ⇒ zk-SNARK)

Tx: proof π , AcmeCo: $\mathbf{com_1}$ → Alice: $\mathbf{com_2}$, AcmeCo: $\mathbf{com_3}$

Miners: accept Tx if proof π is valid (need fast verification)

⇒ learn Tx is valid, but amounts are hidden

Optimized proof?

circuit C(x,w) outputs 0 if:

(i) $com_i = commit(m_i, r_i)$,

~~(ii) $m_1 = m_2 + m_3 + Fees$,~~

(iii) $m_2 \geq 0$ and $m_3 \geq 0$

Easy to check with Pedersen commitment:

set $com \leftarrow com_1/com_2 \cdot com_3 \cdot g^{Fees}$

(a commitment to $m_1 - m_2 - m_3 - Fees$)

prove that $com = commit(0, r)$

remaining proof is ≈400 bytes

(CT is the beginning of MimbleWimble implemented in the Grin blockchain)

Next segment: anonymous payments

Anonymous Payments: Tornado Cash and Zcash / IronFish

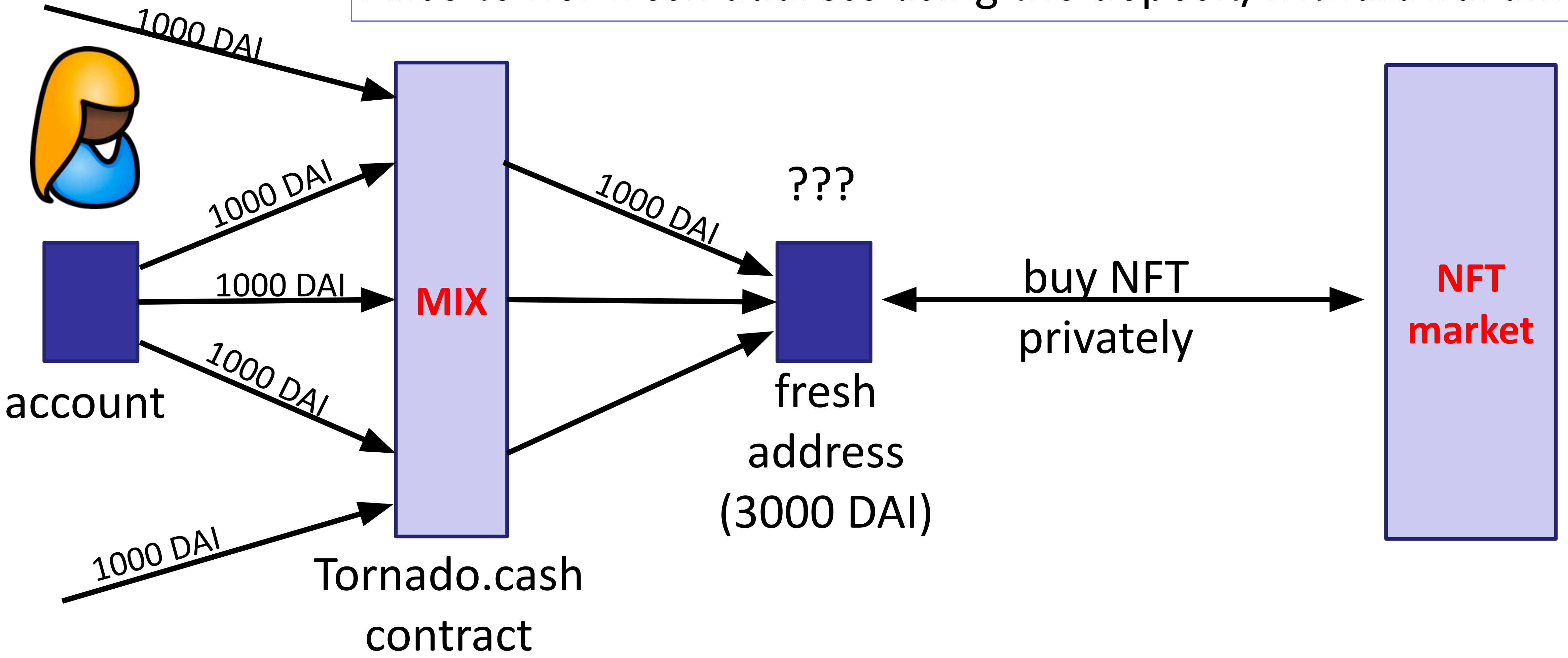
https://defi-learning.org/

TORNADO CASH: A ZK-BASED MIXER

Launched on the Ethereum blockchain on May 2020 (v2)

Tornado Cash: a ZK-mixer

A common denomination (1000 DAI) is needed to prevent linking Alice to her fresh address using the deposit/withdrawal amount

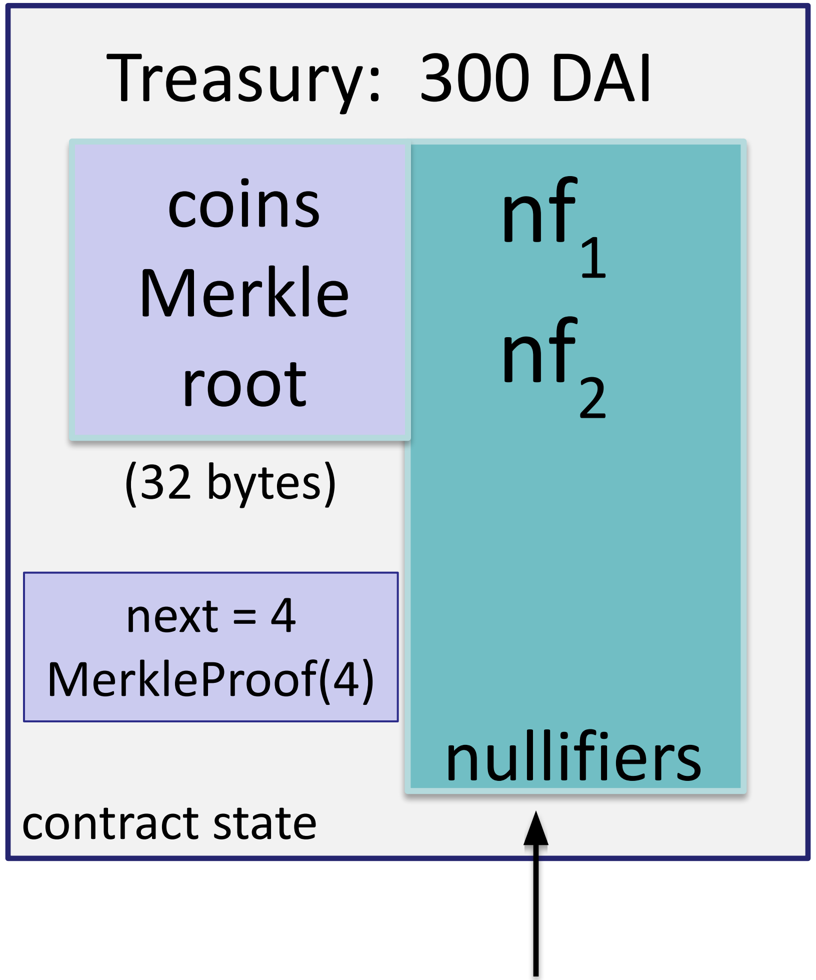


The tornado cash contract (simplified)

100 DAI pool:
each coin = 100 DAI

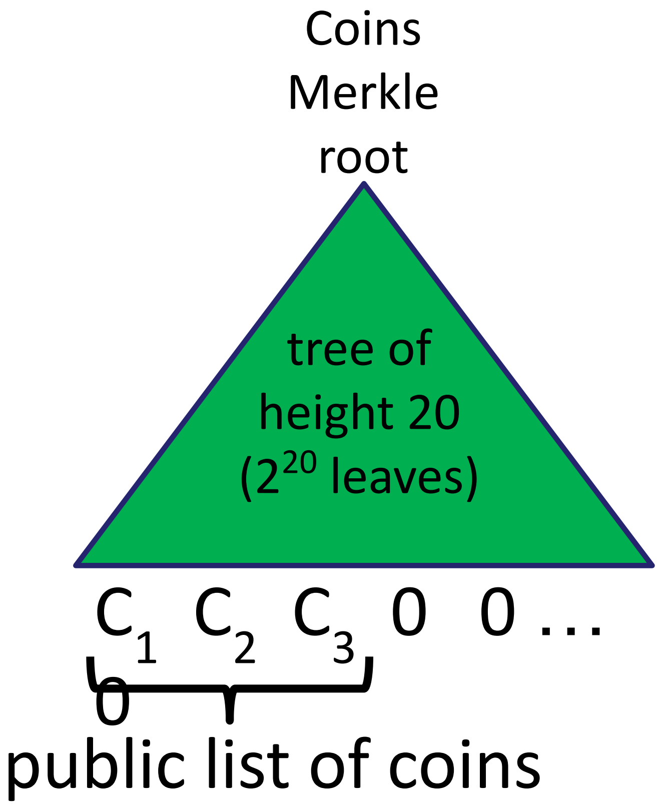
Currently:

- three coins in pool
- contract has 300 DAI
- two nullifiers stored



explicit list:
one entry per **spent coin**

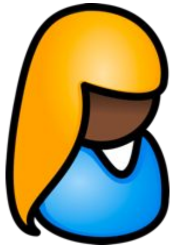
$H_1, H_2: R \rightarrow \{0,1\}^{256}$



Tornado cash: deposit (simplified)

<u>100 DAI pool</u>:
each coin = 100 DAI

<u>Alice deposits 100 DAI:</u>



choose random k, r in R
set $C_4 = H_1(k, r)$
write C_4 in leaf #4 in tree
π = MerkleProof(5)

Treasury: 300 DAI

coins Merkle root
(32 bytes)

next = 4
MerkleProof(4)

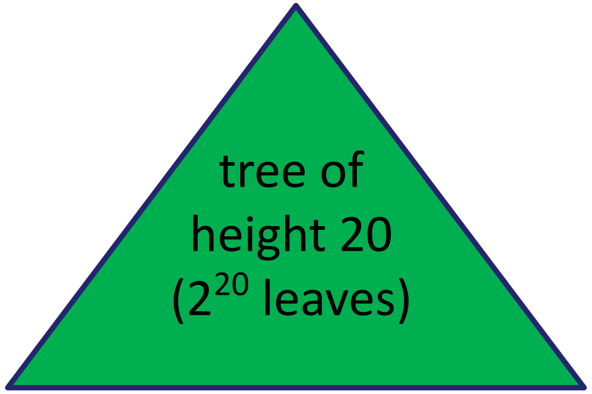
nf_1
nf_2
nullifiers

contract state

explicit list:
one entry per **spent coin**

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

Coins Merkle root

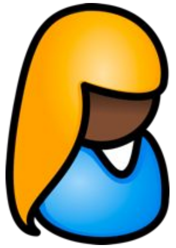


C_1 C_2 C_3 0 0 …
0
public list of coins

Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:



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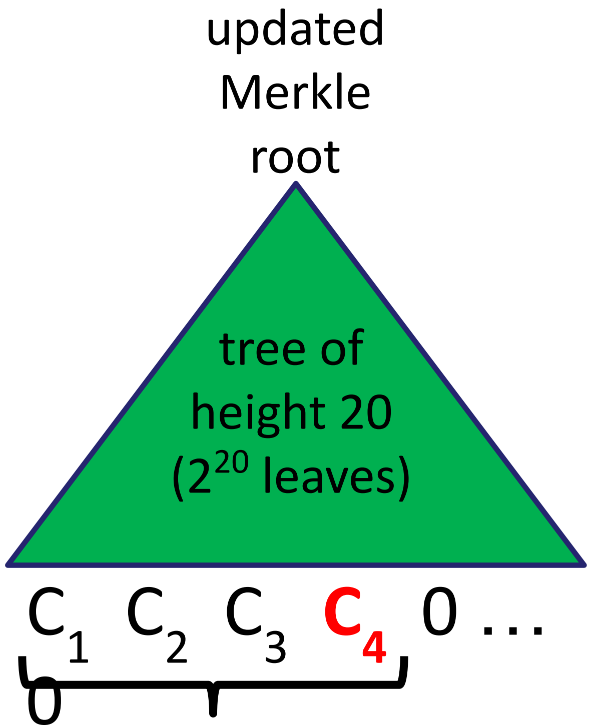
one entry per **spent coin**

H_1, H_2: R → $\{0,1\}^{256}$



public list of coins

Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:

100 DAI

C_4 , MerkleProof(5)

choose random k, r in R
set $C_4 = H_1(k, r)$
write C_4 in leaf #4 in tree
π = MerkleProof(5)

Treasury: 300 DAI

coins Merkle root

(32 bytes)

next = 4
MerkleProof(4)

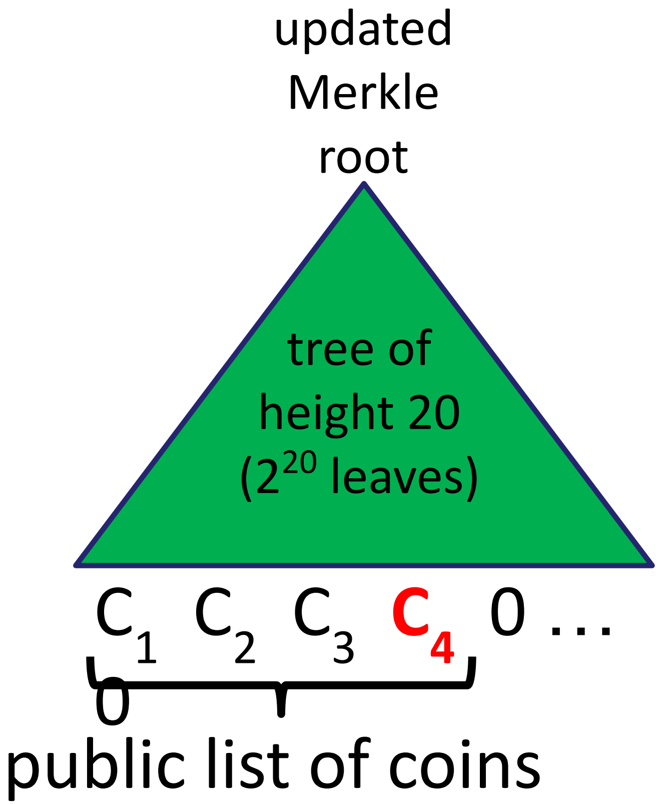
nf_1
nf_2

nullifiers

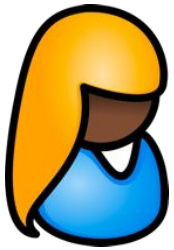
contract state

explicit list:
one entry per **spent coin**

$H_1, H_2: R \rightarrow \{0,1\}^{256}$



Tornado cash: deposit (simplified)



100 DAI

C_4 , π = MerkleProof(5)

coins Merkle root

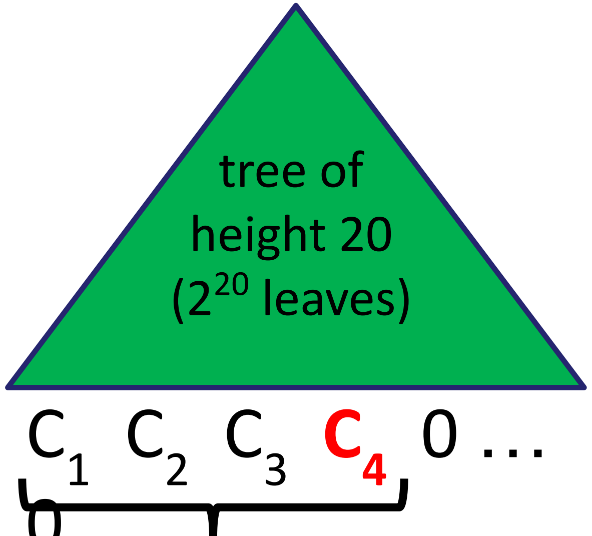
(32 bytes)

next = 4
MerkleProof(4)

Tornado contract

H_1, H_2: R → $\{0,1\}^{256}$

updated Merkle root



public list of coins

Tornado contract does:

(1) use C_4 and MerkleProof(4) to compute updated Merkle root

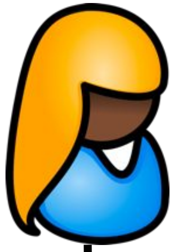
(2) verify π = MerkleProof(5)

(3) if valid: update state

Tornado cash: deposit (simplified)

100 DAI pool:
each coin = 100 DAI

Alice deposits 100 DAI:



100 DAI →
C_4 , MerkleProof(5)

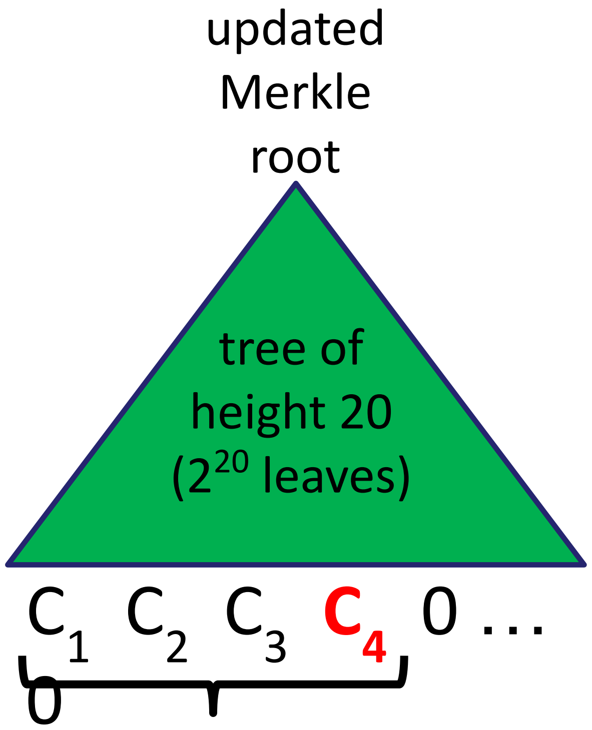
note: (k, r)
Alice keeps secret
(one note per coin)

Treasury: **400** DAI

updated Merkle root (32 bytes)	nf_1 nf_2 nullifiers
next = **5** MerkleProof(**5**)	

updated contract state

Every deposit: new Coin added sequentially to tree

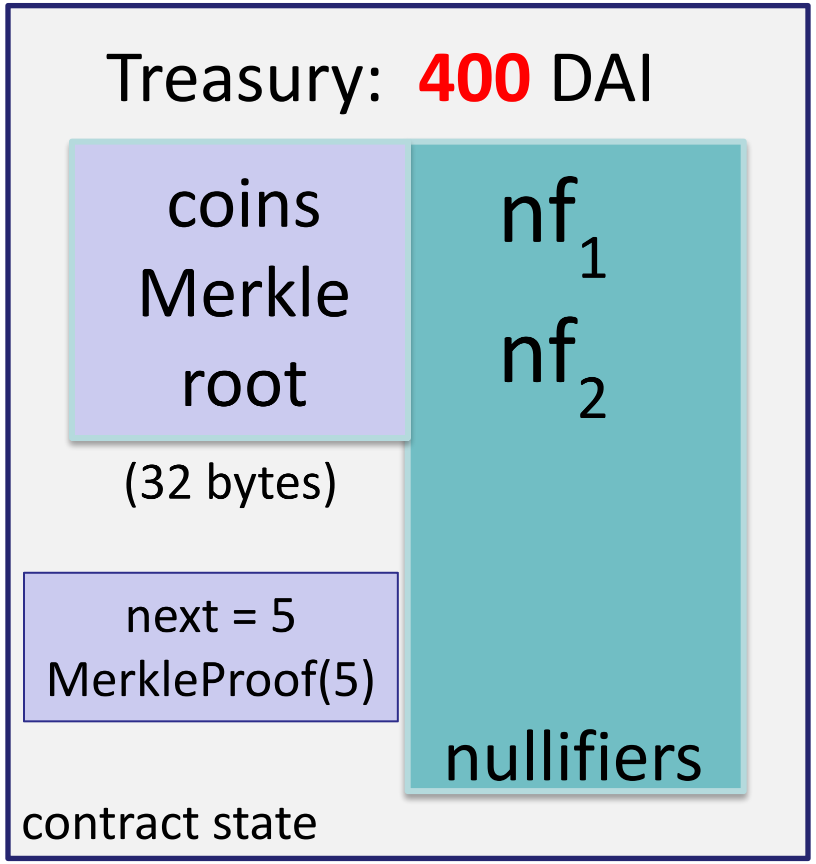


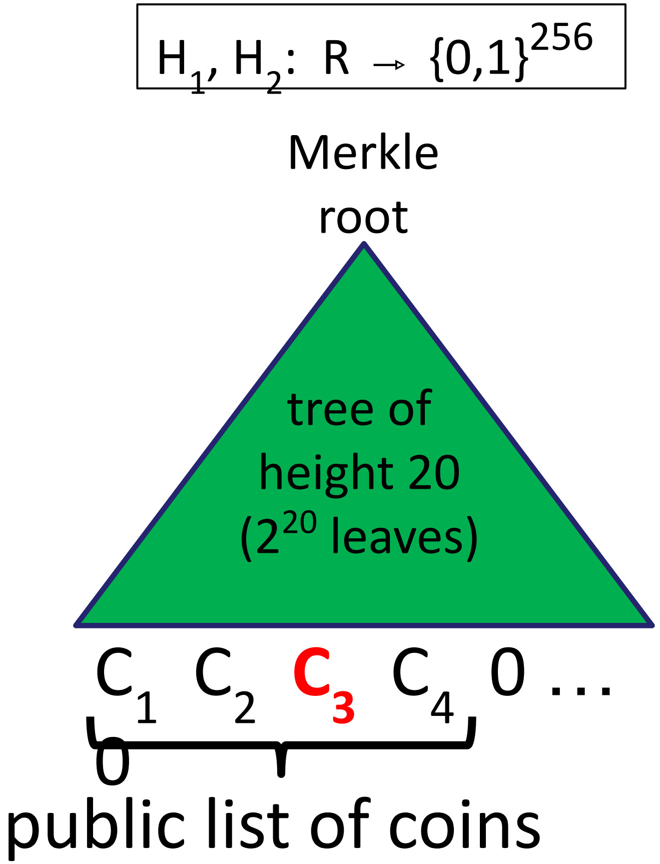
public list of coins

an observer sees who owns which coins

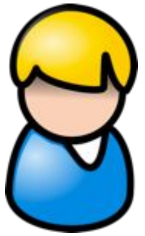
Tornado cash: withdrawal (simplified)

100 DAI pool:
each coin = 100 DAI





Withdraw coin #3 to addr A:



has note= (k′, r′)

set **nf** = H_2(k′)

Bob proves "I have a note for some leaf in the coins tree, and its nullifier is **nf**"
(without revealing which coin)

Tornado cash: withdrawal (simplified)

H_1, H_2: $R \rightarrow \{0,1\}^{256}$

Withdraw coin #3 to addr A:

has note= (k', r')   set **nf** = H_2(k')

Bob builds zk-SNARK proof π for

public statement x = (**root**, **nf**, **A**)

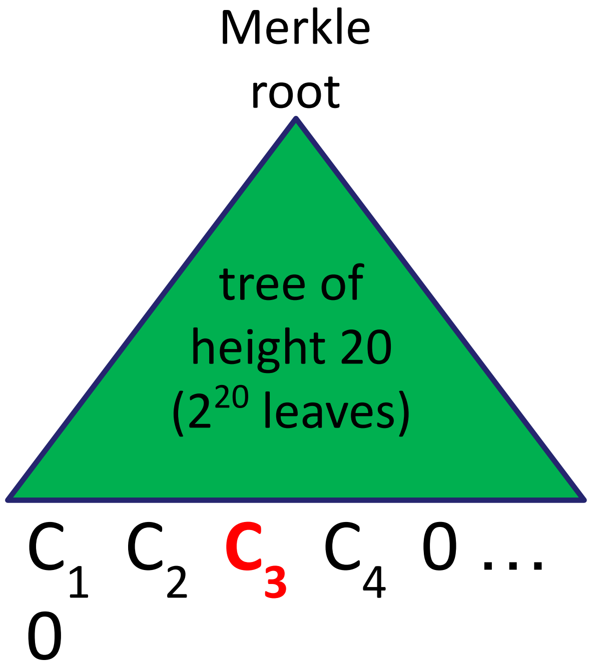
secret witness w = (k', r', C_3, MerkleProof(C_3))

where Circuit(x,w)=0 iff:

(i) C_3 = (leaf #3 of **root**), i.e. MerkleProof(C_3) is valid,

(ii) $C_3 = H_1$(k', r'), and

(iii) **nf** = H_2(k').



(address A not used in Circuit)

Tornado cash: withdrawal (simplified)

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

Withdr

h

The address A is part of the statement to ensure that a miner cannot change A to its own address and steal funds

Assumes the SNARK is non-malleable:

adversary cannot use proof π for x to build a proof π' for some "related" x'
(e.g., where in x' the address A is replaced by some A')

Bob builds zk-SNARK proof π for

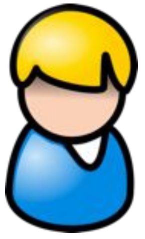
public statement x = (**root**, **nf**, **A**)

secret witness w = (k', r', C_3, MerkleProof(C_3))

Tornado cash: withdrawal (simplified)

100 DAI pool:
each coin = 100 DAI

Withdraw coin #3 to addr A:



nf, proof π, **A** →
(over Tor)

Bob's ID and coin C_3 are not revealed

Treasury: **400** DAI

coins Merkle root
(32 bytes)

next = 5
MerkleProof(5)

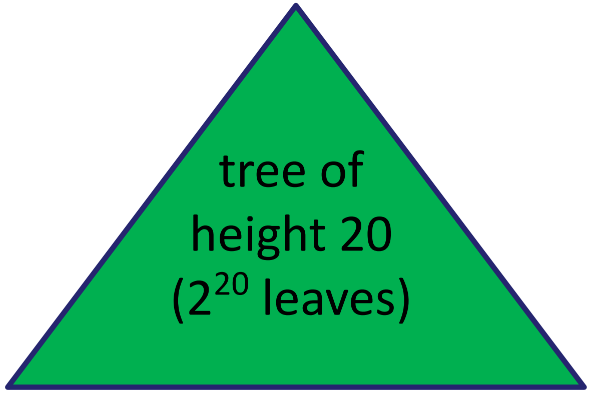
nf_1
nf_2

nullifiers

contract state

$H_1, H_2: R \rightarrow \{0,1\}^{256}$

Merkle root



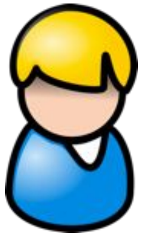
C_1 C_2 C_3 C_4 0 …
0
public list of coins

Contract checks (i) proof π is valid for (root, **nf**, **A**), and (ii) **nf** is not in the list of nullifiers

Tornado cash: withdrawal (simplified)

<u>100 DAI pool</u>:
each coin = 100 DAI

<u>Withdraw coin #3 to addr A:</u>



nf, proof π, **A** →
(over Tor)

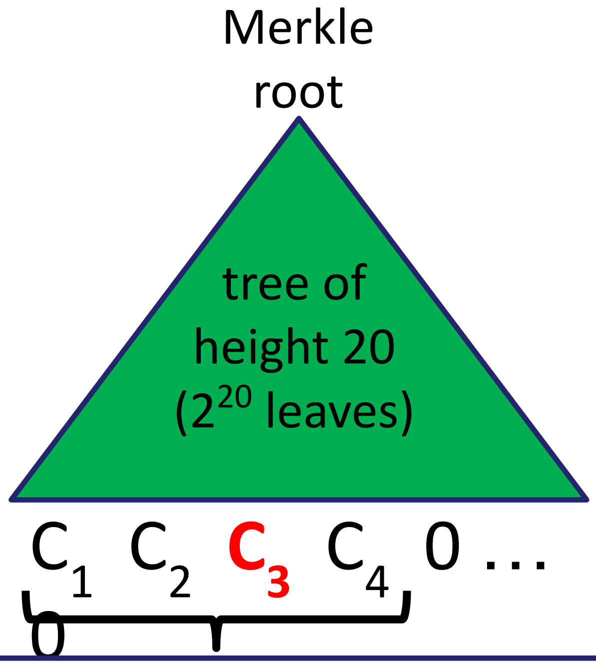
← **100 DAI**
to address **A**

Treasury: **300** DAI

coins Merkle root	nf_1 nf_2 **nf**
(32 bytes)	
next = 5 MerkleProof(5)	nullifiers

contract state

H_1, H_2: $R \rightarrow \{0,1\}^{256}$



public list of coins
… but observer does not know which are spent

nf and π reveal nothing about which coin was spent.

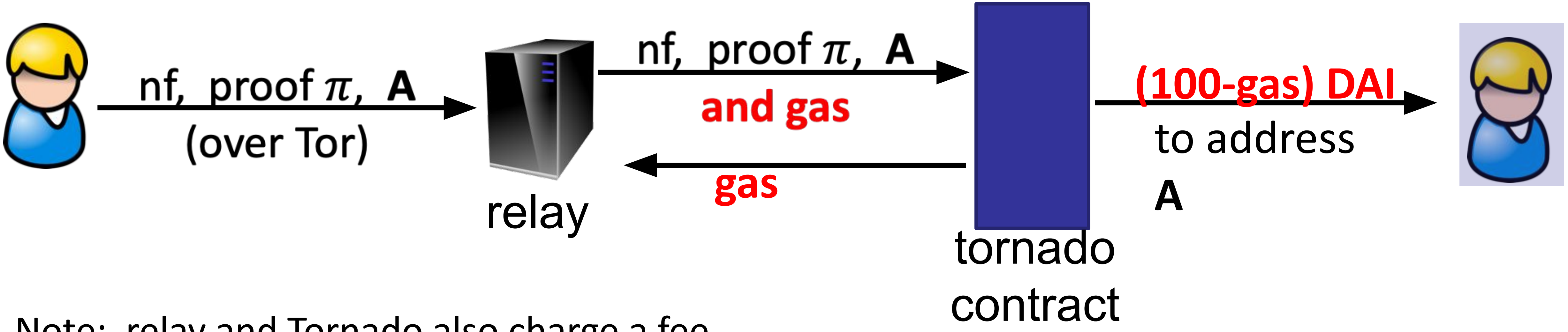
But, coin #3 cannot be spent again, because **nf = H_2(k')** is now nullified.

Who pays the withdrawal gas fee?

Problem: how does Bob pay for gas for the withdrawal Tx?

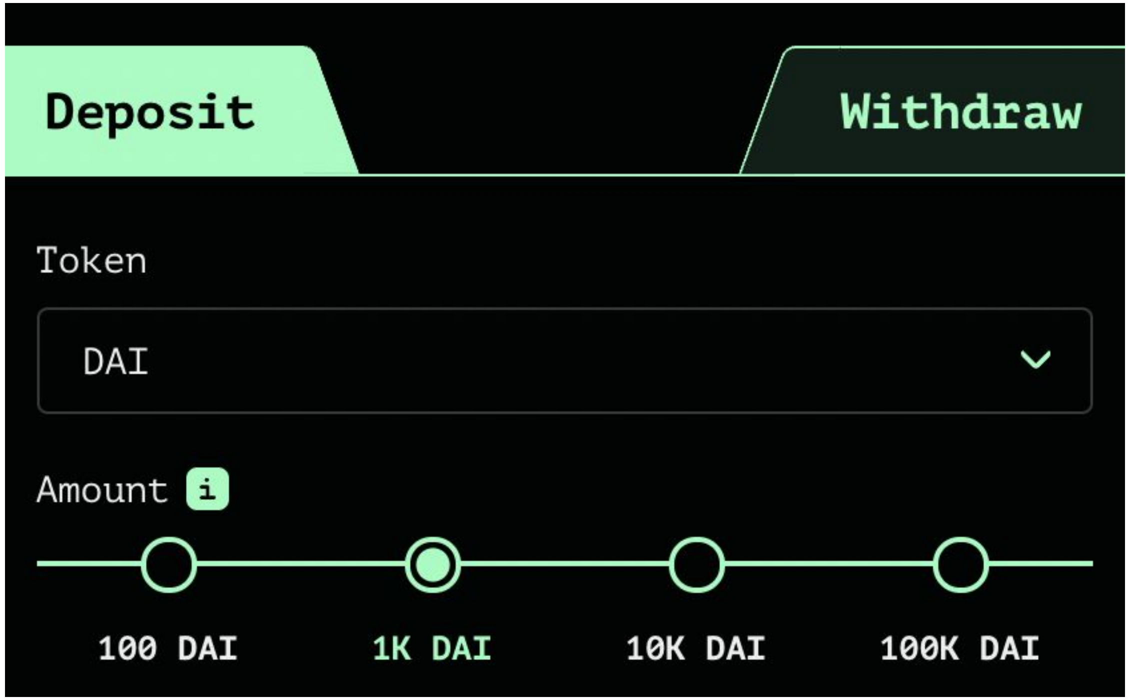
- If paid from Bob's address, then fresh address is linkable to Bob

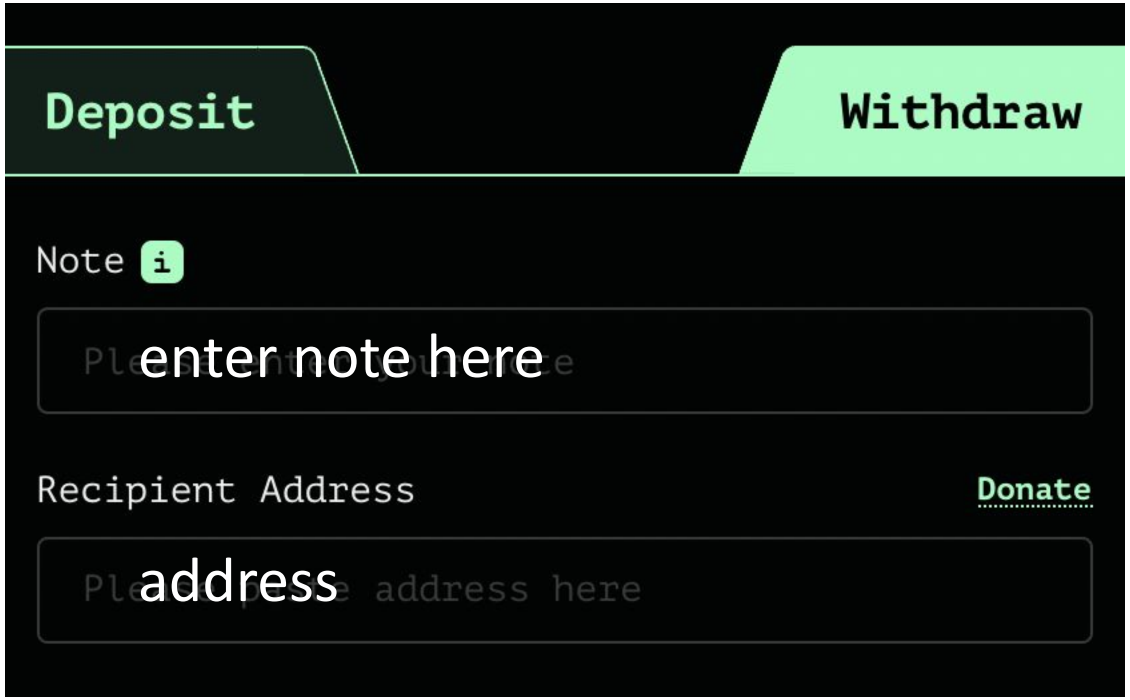
Tornado's solution: **Bob uses a relay**



Note: relay and Tornado also charge a fee

Tornado Cash: the UI





After deposit: get a note

Later, use note to withdraw

(wait before withdrawing)

Anonymity set

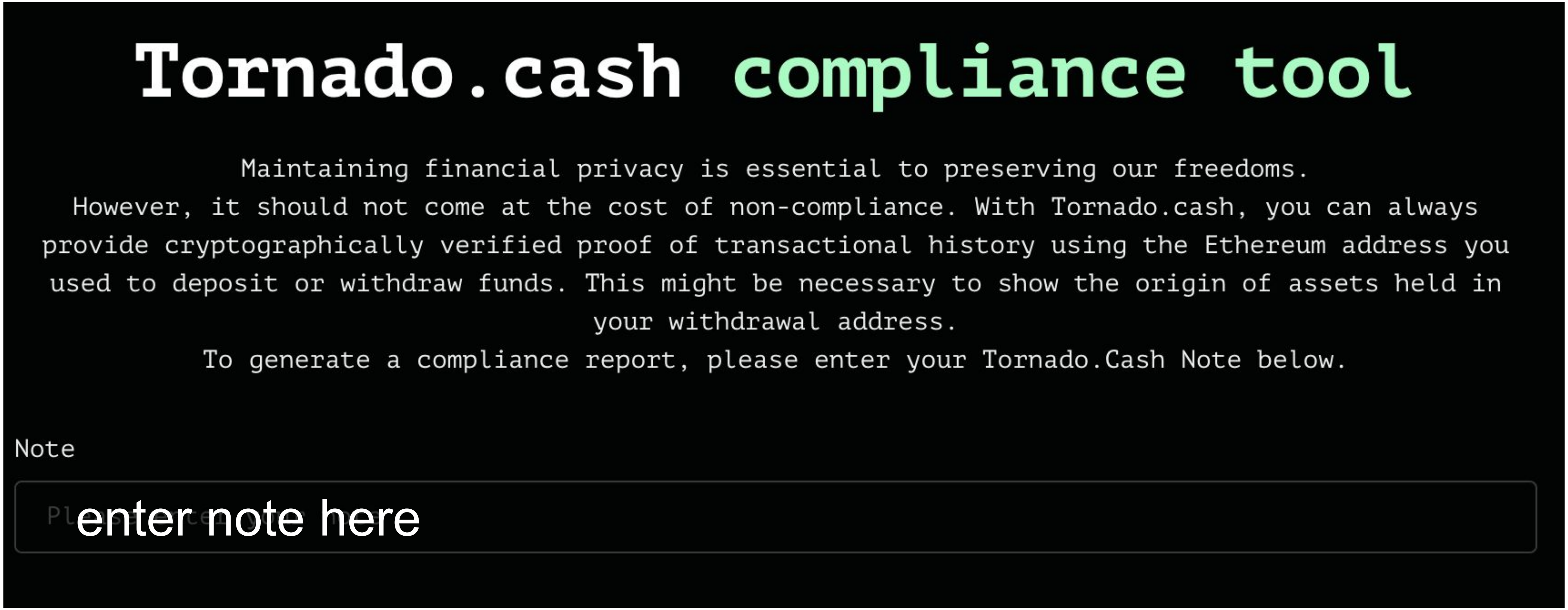
88,036
Total deposits

leaves occupied
over all pools

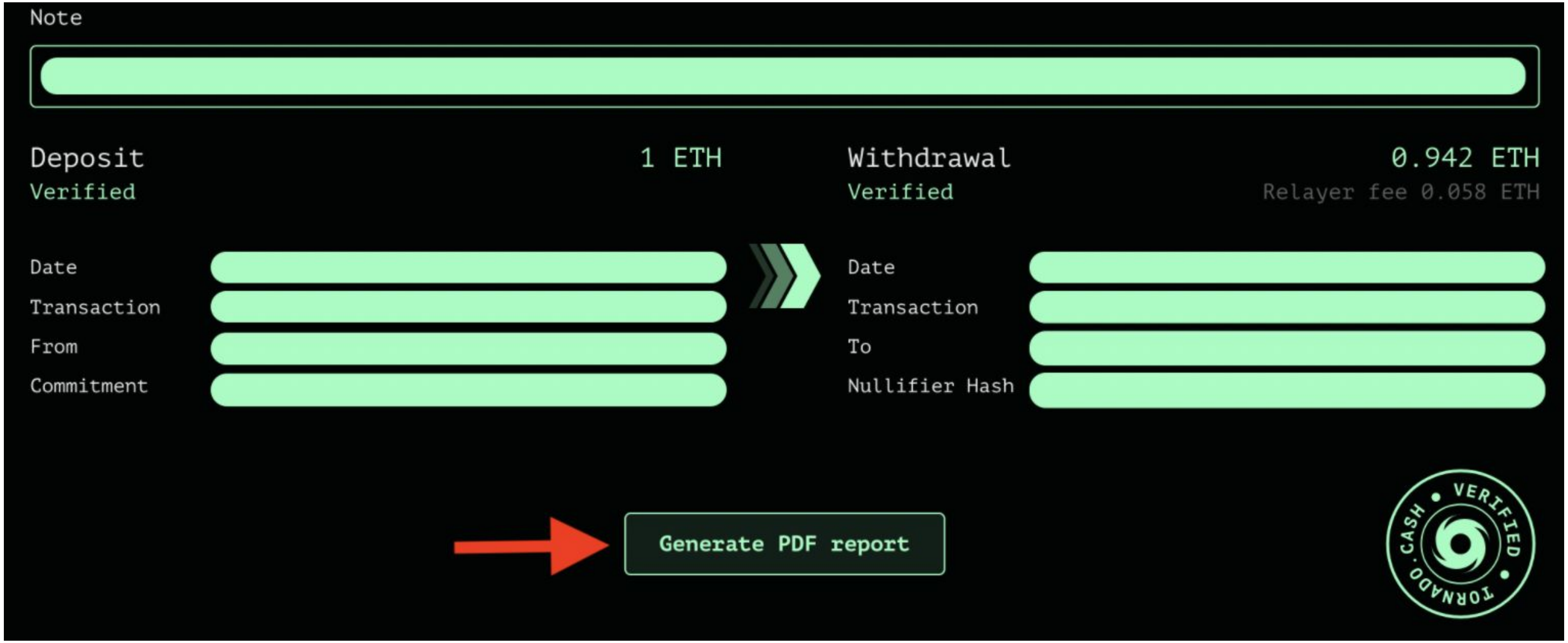
$3,798,916,834
Total USD deposited

Oct. 2021

Compliance tool



Compliance tool



Reveals source address and destination address of funds

ZCASH / IRONFISH

Two L1 blockchains that extend Bitcoin.

Sapling (Zcash v2) launched in Aug. 2018.

More complicated, but similar use of Nullifiers

Zcash / IronFish (simplified)

Goal: fully private payments … like cash, but across the Internet

Includes mechanisms to let parties abide by financial regulation

Zcash / IronFish supports two types of TXOs:

- **transparent** (as in Bitcoin)
- **shielded** (anonymized)

a Tx can have both types of inputs, both types of outputs

Addresses and coins (notes)

H_1, H_2, H_3: cryptographic hash functions.

sk needed to spend note for address pk

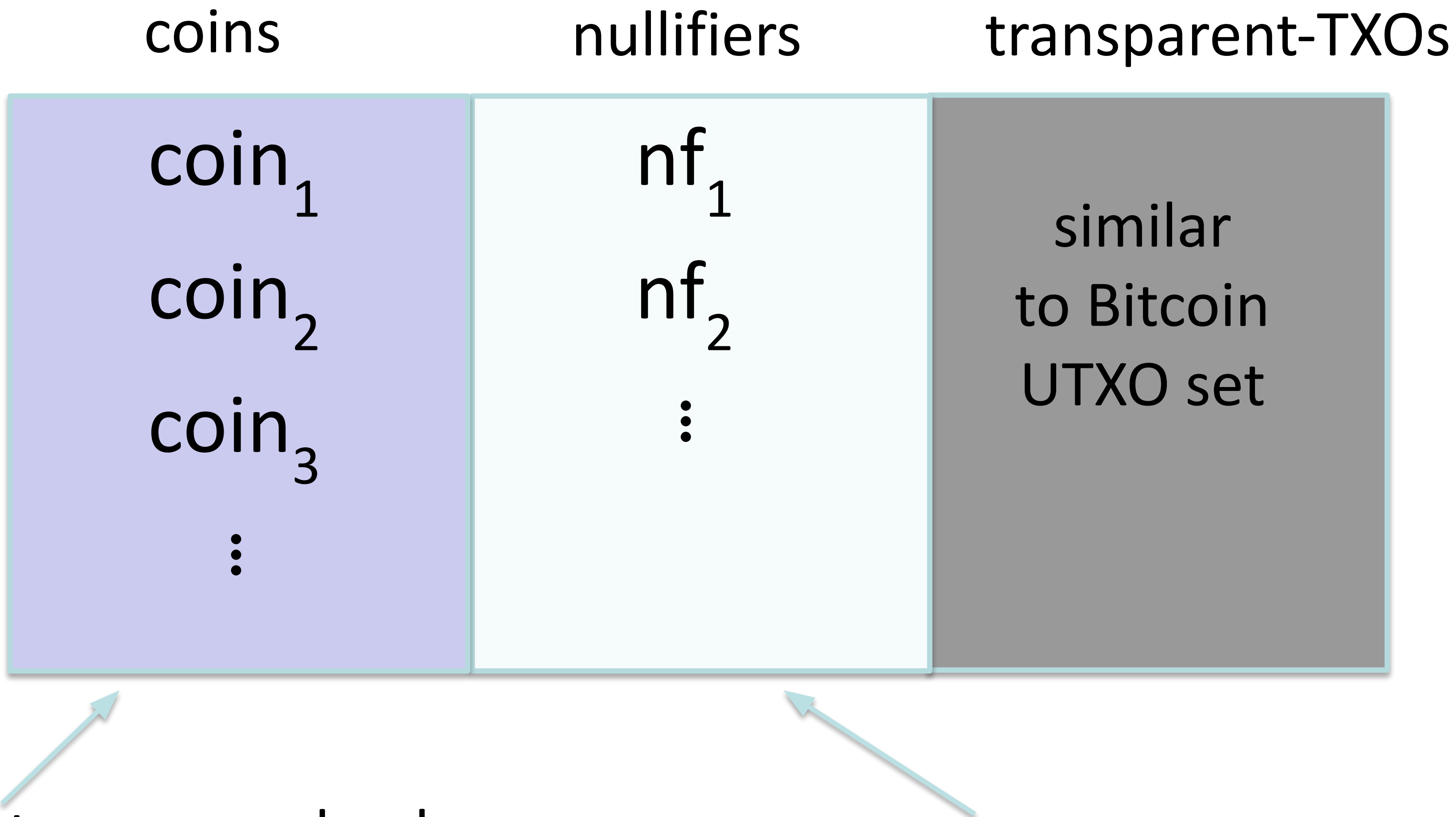
(1) shielded address: random $sk \leftarrow X$, $pk = H_1(sk)$

(2) **shielded coin** owned by address pk:

- coin owner has (from payer): value v and $r \leftarrow R$
- on blockchain: $coin = H_2(\ (pk, v)\ ,\ r)$ (commitment to pk, v)

pk: addr. of owner, v: value of coin, r: random chosen by payer

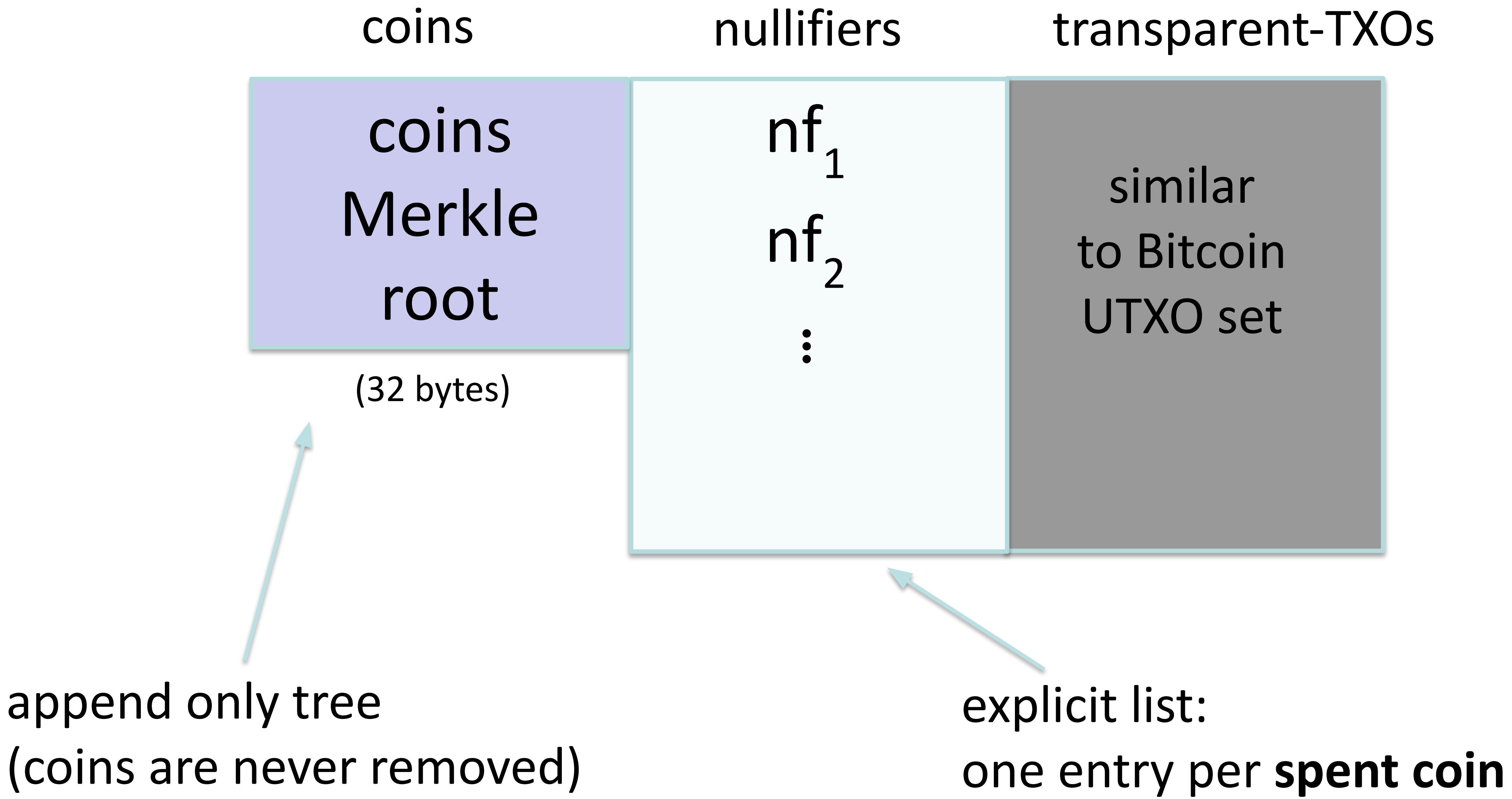
The blockchain



just Merkle root … append only tree
(coins are never removed)

explicit list:
one entry per **spent coin**

The blockchain



Transactions: an example

owner of **coin** = H_2((pk, v) , r)

wants to send **coin** value v to: [shielded pk', v'
transp. pk'', v'']

(v = v' + v'')

---

step 1: construct new coin: **coin'** = H_2((pk', v') , r')

by choosing random r' ← R (and send (v', r') to owner of pk')

step 2: compute **nullifier** for spent coin **nf** = H_3(sk, index of coin in Merkle tree)

nullifier **nf** is used to "cancel" **coin** (no double spends)

key point: miners learn that some coin was spent, but not which one!

Transactions: an example

step 3: construct a zk-SNARK proof π for

statement = x = (current Merkle root, **coin'**, **nf**, v'')

witness = w = (sk, (v, r), (pk', v', r'), MerkleProof(**coin**))

C(x, w) outputs 0 if: compute **coin** := H_2((pk=H_1(sk), v), r) and check

The Zcash circuit

(1) MerkleProof(**coin**) is valid,

(2) **coin'** = H_2((pk', v') , r')

(3) v = v' + v'' and v' ≥ 0 and v'' ≥ 0

(4) **nf** = H_3(sk, index-of-coin-in-Merkle-tree)

from Merkle proof

What is sent to miners

step 4: send (**coin'**, **nf**, transparent-TXO, proof π) to miners,

send (v' , r') to owner of pk'

step 5: miners verify

(i) proof π and transparent-TXO

(ii) verify that **nf** is not in nullifier list (prevent double spending)

if so, add **coin'** to Merkle tree, add **nf** to nullifier list,

add transparent-TXO to UTXO set.

Summary

- Tx hides which coin was spent

  ⇒ **coin** is never removed from Merkle tree,
  but cannot be double spent thanks to nullifer

  note: prior to spending **coin**, only owner knows **nf**:

  $\mathbf{nf} = H_3(sk,$ index of coin in Merkle tree $)$

- Tx hides address of **coin’** owner

- Miners can verify Tx is valid, but learns nothing about Tx details

End of lecture.   Let's do a quick review.

A zk-SNARK for a circuit C:

- Given a public statement x, prover P outputs a proof π that "convinces" verifier V that prover knows w s.t. $C(x, w) = 0$.
- Proof π is short and fast to verify

What is it good for?

- Private payments and private Dapp business logic (Aleo)
- Private compliance and L2 scalability with privacy

More to think about:

- private DAO participation? private governance?

Further topics (see, e.g., cs251)

- How to build a zk-SNARK?
- Recursive SNARKs:
  - Proving knowledge of a SNARK proof

    1-level recursive statement: "I know a proof π that $\exists w : C(x, w) = 0$"
  - Used in systems that keep business logic private
- Privately communicating with the blockchain: Nym
  - And (privately) compensating proxies for relaying traffic

END OF TOPIC

https://defi-learning.org/